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Effects of energy chirp on echo-enabled harmonic generation free-electron lasers*

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Abstract

We study effects of energy chirp on echo-enabled harmonic generation (EEHG). Analytical expressions are compared with numerical simulations for both harmonic and bunching factors. We also discuss the EEHG free-electron laser bandwidth increase due to an energy-modulated beam and its pulse length dependence on the electron energy chirp.

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1 Introduction

A new method of harmonic generation using the beam echo effect is proposed in [1] and is studied in details in [2]. The echo-enabled harmonic generation (EEHG) has a remarkable up-frequency conversion efficiency and may drastically simplify the design of seeded, short-wavelength free-electron lasers (FELs). It is largely anticipated that a seeded FEL will be used for the production of relatively long temporally coherent radiation pulses with a narrow bandwidth. This typically requires a very uniform electron bunch with constant current and energy, and specific designs are proposed to generate such a bunch in the accelerator [3]. Nevertheless, residual energy variations due to nonlinearity of the machine or energy modulations due to microbunching instability will be unavoidable and may broaden the bandwidth of such a seeded FEL. In addition, an electron beam with a large energy chirp may be also useful to overcome the sensitivity of the seeded FEL power to electron energy jitters or to control the FEL pulse length. In this paper, we study the dependence of the echo microbunching on the initial energy chirp of the electron beam. We also discuss the EEHG FEL performance in presence of a chirped electron beam or any residual energy modulation created from the upstream accelerator.

2 Echo microbunching in presence of a linear chirp

We consider a linearly chirped electron beam with the initial longitudinal distribution function as

$$f_0(p,\zeta) = \frac{N_0}{\sqrt{2\pi}} \exp\left[\frac{-(p-h\zeta)^2}{2}\right] , \tag{1}$$

where N_0 is the number of electrons per unit length, $p = (E - E_0)/\sigma_E$, E_0 is the beam central energy, σ_E is the initial uncorrelated energy spread, $\zeta = k_1 z$, and the chirp is defined by

$$h = \frac{dp}{d\zeta} = \frac{d\delta}{dz}\frac{1}{k_1\sigma_E/E_0} . \tag{2}$$

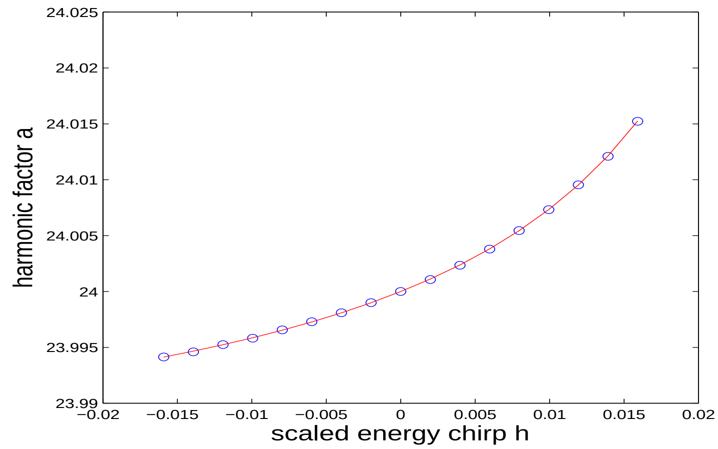
Following the notation and the derivation in Appendix A of Ref. [2], the bunching factor at the harmonic factor a after double modulators and double chicanes is

$$b = \frac{1}{N_0}\left|\int_{-\infty}^{\infty} dp e^{-iapB} f_0(p,\zeta)\langle e^{-ia\zeta} e^{-iaA_1 B\sin\zeta} e^{-iaA_2B_2\sin(K\zeta+KB_1p+KA_1B_1\sin(\zeta+\phi))}\rangle\right| , \tag{3}$$

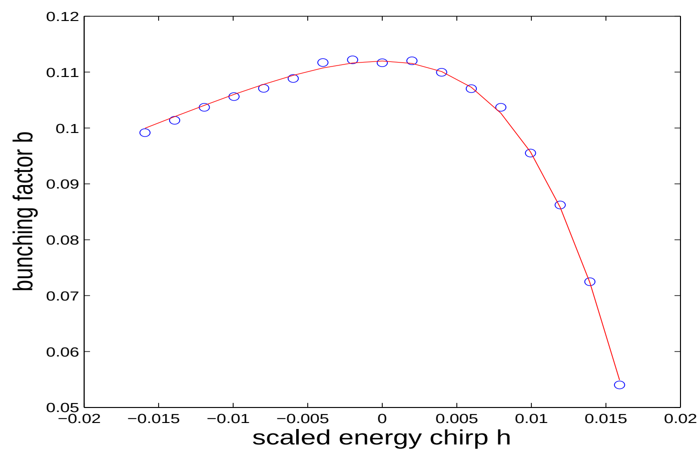
where $k_{1,2}$ are the first and the second laser wavenumbers, $K = k_2/k_1$, $B_{1,2} = R_{56}^{(1,2)} k_1\sigma_E/E_0$, $B = B_1 + B_2$, $A_{1,2} = \Delta E_{1,2}/\sigma_E$, and the average is carried out over ζ for the periodic modulation.

By changing the integration variable from p to $p' = p - h\zeta$, the averaging over ζ can be carried out to obtain the non-vanishing harmonic factor at

$$a = \frac{n + mK(1+hB_1)}{1+hB} , \tag{4}$$



(a) Harmonic factor.



(b) Bunching factor.

Figure 1: Echo harmonic and bunching factors vs. energy chirp for fixed chicane strengths.

and the bunching factor is

$$
\begin{aligned}
b &= \left| J_m(-aA_2B_2) J_n\left(A_1(mKB_1 - aB)\right) \right| \exp\left[\frac{-(aB - mKB_1)^2}{2}\right] \\
&= \left| J_m\left(\frac{n + mK(1 + hB_1)}{1 + hB} A_2B_2\right) J_n\left(\frac{A_1(nB + mKB_2)}{1 + hB}\right) \right| \exp\left[\frac{-(nB + mKB_2)^2}{2(1 + hB)^2}\right] . \quad (5)
\end{aligned}
$$

The largest bunching factor is obtained when $n = \pm 1$ and we take $n = -1$ and m positive in order to use two chicanes with the same signs of R_{56}. After this simplification, the bunching factor becomes

$$
b = \left| J_m\left(\frac{-1 + mK(1 + hB_1)}{1 + hB} A_2B_2\right) J_1\left(\frac{A_1(mKB_2 - B)}{1 + hB}\right) \right| \exp\left[\frac{-(mKB_2 - B)^2}{2(1 + hB)^2}\right] , \quad (6)
$$

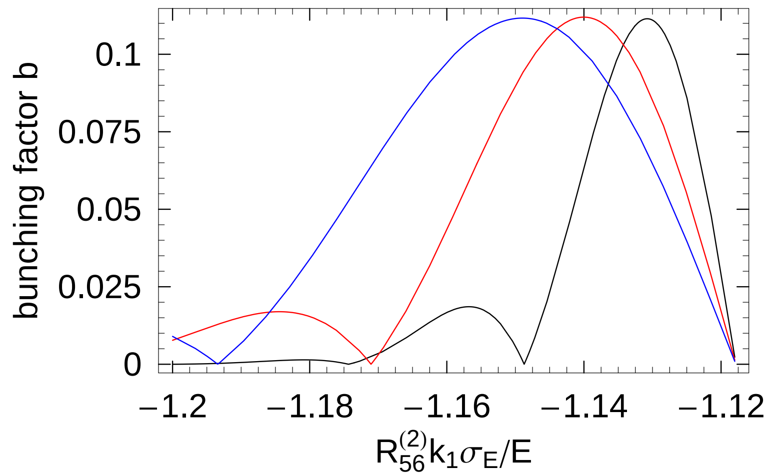


Figure 2: Bunching factor b determined from Eq. (6) vs the second chicane strength B_2 for $h = 0$ (red), $h = -0.15$ (blue) and $h = 0.15$ (black).

and the echo harmonic factor is

$$a = \frac{mK(1 + hB_1) - 1}{1 + hB}\,. \tag{7}$$

We carry out one-dimensional echo simulations based on numerical examples given in Ref. [2] and compare with Eqs. (6) and (7). The parameters were chosen following the FERMI FEL [4] design goal of reaching 10 nm radiation wavelength from 240 nm ultraviolet laser using 1.2 GeV beam with 150 keV rms slice energy spread. Here $A_1 = 3$, $B_1 = -26.828$, $A_2 = 1$, $B_2 = -1.140$, $K = 1$, which has been optimized for the harmonic factor $a = 24$. Figures 1(a) and 1(b) show the excellent agreement between theory and simulations. Note that $h = 0.015$ corresponds to a relative energy chirp of ~ 50 m^{-1}. For a chirped beam, the bunching factor is degraded from the maximum value optimized for $h = 0$, but a slight change of B_2 can restore the maximum bunching for a given h (see Fig. 2). In passing, we note that the bunch length will be compressed by a factor $1/(1 + hB)$ after the double chicanes.

3 Effects on EEHG FELs

One important goal for seeded FELs is generation of Fourier transform limited pulses with a very narrow-bandwidth. In this case, the electron energy profile must be made very flat and well within the FEL gain bandwidth. Nevertheless, residual energy modulation due to microbunching instability may still change the echo frequency and dictate the FEL bandwidth. We can estimate the frequency shift by expanding Eq. (7) for $|h| \ll 1$ to first order as

$$a \approx (mK-1) + (B - mKB_2)h\,. \tag{8}$$

Note that the factor $(B-mKB_2)$ also appears in Eq. (6). In order to maximize the bunching factor, $(B-mKB_2) \approx 1$ for $A_1 \leq 1$ and drops to about 0.5 for $A_1 = 3$. Since this factor is relatively insensitive to A_1, we have

$$(\Delta a)_{\text{echo}} \approx \frac{\Delta h}{2}\,. \tag{9}$$

Thus, for a bunch with variable chirps due to nonlinear energy modulation, the bandwidth increase of echo microbunching is approximately one half of the scaled chirp variation defined as in Eq. (2), independent of the strengths of both chicanes. We continue with the FERMI FEL example given in the previous section. Suppose a flat bunch of ~ 100 μm is used to generate a Fourier transform limited bandwidth of $\sim 10^{-4}$ at 10-nm radiation wavelength, the required chirp variation should be $\Delta h < 2a \times 10^{-4} \approx 5 \times 10^{-3}$. This corresponds to a energy variation less than the slice energy spread within any 8 μm section of the bunch. We note that undesirable density modulations converted from these energy variations may also increase the FEL bandwidth and is not taken into account here.

We can compare the echo bandwidth increase to a similar effect in high-gain harmonic generation (HGHG) FELs when nonlinear energy chirp is taken into account. After a chirped electron beam is energy-modulated at the seed wavenumber k_1, the buncher chicane will shift the n^{th} harmonic microbunching wavenumber by [5]

$$\Delta k_n \approx n k_1 \frac{d\delta}{dz} R_{56}^{(1)}\,. \tag{10}$$

For HGHG, the chicane strength is typically optimized to maximize the harmonic bunching as

$$R_{56}^{(1)} \approx \frac{1}{k_1 \Delta E/E_0} \approx \frac{1}{k_1 n \sigma_E/E_0}\,, \tag{11}$$

where $\Delta E \approx n\sigma_E$ is the required energy modulation to reach the n^{th} harmonic. For a beam with nonlinear energy modulation, this will determine the final bandwidth as

$$(\Delta a)_{\text{HGHG}} = \frac{\Delta k_n}{k_1} \approx \Delta h\,. \tag{12}$$

Assuming that a cascaded HGHG will reach the same final harmonic factor a as EEHG, we find that the effect of nonlinear energy modulation on the EEHG bandwidth is quite similar to that on the HGHG bandwidth.

In addition, a linearly chirped beam may be useful to control the seeded FEL pulse duration since only part of electrons having energies within the gain bandwidth will contribute to the FEL interaction. Although the FEL pulse duration can in principle be controlled by the seed laser, it may be operationally convenient to control the x-ray pulse length by varying the electron energy chirp using the linac. Assuming that the seed laser pulse covers the entire electron bunch length, an estimation of the rms FEL pulse duration can be made as [6]

$$\sigma_t \approx \frac{\sigma_\nu}{|u|}\,, \tag{13}$$

where σ_ν is the relative FEL gain bandwidth, and

$$u = 2ck_1 \frac{h}{1+hB}\frac{\sigma_E}{E_0} \tag{14}$$

is the resulting resonant frequency chirp due to energy chirp after the beam passing the double chicanes. Let us demonstrate the short pulse possibilities of a chirped EEHG FEL with a numerical example below (and without full system optimization). We assume an initial energy chirp of 1% over 60 μm bunch length can be generated in the accelerator (i.e., $h = 0.05$). For this large energy chirp, B_1 (and B_2) should be reduced from the previous numerical example so that the bunch is not over-compressed after the double chicanes. This can be achieved by setting $A_2 = A_1 = 3$, then the optimal chicane strengths are $B_1 = -8.59$ and $B_2 = -0.371$ for the same final echo microbunching wavelength ($\sim$ 10 nm). After passing the double chicane, the bunch will be compressed by a factor of $1/(1 + hB) \approx 1.8$. Since the typical rms bandwidth of a soft x-ray FEL is $\sigma_\nu \sim 10^{-3}$, we expect from Eq. (13) that the chirped electron beam will generate $\sim$ 5 fs (rms) saturated radiation pulses in a 10-nm FEL undulator.

4 Conclusions

In this paper, we study effects of energy chirp on echo microbunching and EEHG FELs. Despite that much stronger chicanes are used in EEHG than in the standard HGHG, the FEL bandwidth increase associated with the residual energy modulation from the upstream accelerator is comparable in both schemes. Thus, accelerator designs that control the uniformity of beam longitudinal phase space for HGHG is sufficient for EEHG. Finally, we illustrate the possibilities of generating a few femtosecond soft x-ray pulse with a chirped EEHG FEL.

5 Acknowledgements

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